

**Algebra**  
**3rd prep**  
**2012-2013**  
**1st - term**

## Cartesian Product

### Sheet (1)

#### **[1] Put (✓) or (x):-**

- (a)  $(2, 3) = (3, 2)$  ( )
- (b)  $(2, 3) = \{2, 3\}$  ( )
- (c) If  $(2, 3) = (\chi, 3)$ , then  $\chi = 2$  ( )
- (d) If  $(a, b) = (\chi, y)$ , then  $a = \chi$ ,  $b = y$  ( )
- (e) If  $a = b$ , then  $(a, b) = (b, a)$  ( )

#### **[2] Find a , b in each of the following:**

(i)  $(a, b) = (-5, 9)$

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(ii)  $(a - 2, b + 1) = (2, -3)$

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(iii)  $(6, b - 3) = (2 - a, -1)$

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(iv)  $(a - 7, 26) = (-2, b^3 - 1)$

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**[3] Complete:**

(a)  $X \times Y = \{ (a, b) : a \in \dots\dots\dots, b \in \dots\dots\dots \}$

(b)  $n(X \times Y) = n(\quad) \times n(\quad)$

(c) If  $(K, m) \in X \times Y$ , then  $K \in \dots\dots\dots, m \in \dots\dots\dots$

(d) If  $X$  is a non – empty set, then:

$$X \times X = \{ (a, b) : a \in \dots\dots\dots, b \in \dots\dots\dots \}$$

(e)  $X \times Y = Y \times X$  when .....

(f) If  $n(X) = 3$ ,  $n(X \times Y) = 6$ , then  $n(Y) = \dots\dots\dots$

(g) If  $X = \{ 2, 4, 6 \}$ ,  $Y = \{ 3, 5, 7, 9 \}$ , then  $n(X \times Y) = \dots\dots\dots$

**[4]** If  $X = \{ 1 \}$ ,  $Y = \{ 2, 3 \}$ ,  $Z = \{ 2, 5, 6 \}$ , represent the sets of  $X$ ,  $Y$ ,  $Z$  with Venn diagram then find:

**a)  $X \times Y$**

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**b)  $Y \times Z$**

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**c)  $X \times Z$**

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d)  $Y^2$

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e)  $(X \times Y) \cup (Y \times Z)$

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f)  $X \times (Y \cap Z)$

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g)  $(X \times Y) \cap (X \times Z)$

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h)  $(Z - Y) \times (X \cup Y)$

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## Sheet (2)

### **First : Complete:**

(1) If  $(a + 5, 3) = (8, b - 1)$  then  $a = \dots\dots\dots$ ,  $b = \dots\dots\dots$

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(2) If  $(\chi^5, y+1) = (32, \sqrt[3]{27})$  then  $\chi = \dots\dots\dots$ ,  $y = \dots\dots\dots$

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(3) If  $(\chi - 1, 11) = (8, y + 3)$ , then  $\sqrt{\chi + 2y} = \dots\dots\dots$

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(4) If  $n(X^2) = 9$ , then  $n(X) = \dots\dots\dots$

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(5) If  $X \times Y = \{(2, 6), (2, 9), (3, 6), (3, 9), (5, 6), (5, 9)\}$ , then....

$x = \dots\dots\dots$ ,  $y = \dots\dots\dots$

### **Second: Choose the correct answer:**

(1) If  $n(X) = 3$ ,  $n(X \times Y) = 12$ , then  $n(Y) = \dots\dots\dots$  (4, 9, 15, 36)

(2) If  $(3, 5) \in \{3, 6\} \times \{X, 8\}$ , then  $X = \dots\dots\dots$  (8, 6, 5, 3)

(3) If the point  $(5, b-7)$  is located on the X – axis then  $b = \dots\dots\dots$

(2, 5, 7, 12)

(4) If the point  $(x-4, 2-x)$  where  $X \in \mathbb{Z}$  is located in the third quadrant,

then  $X = \dots\dots\dots$  (2, 6, 3, 4)

**Third:**

(1) If  $X = \{ 2, 3 \}$ ,  $Y = \{ 3, 4, 5 \}$ , find:

(a)  $X \times Y$

(b)  $n(Y^2)$

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(c)  $n(X \times Y)$

(d)  $(X \times Y) \cap Y^2$

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(2) If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ , find:

(a)  $X, Y$

(b)  $Y \times X$

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(c)  $Y^2$

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(3) If  $X = \{ 3, 4 \}$ ,  $Y = \{ 4, 5 \}$ ,  $Z = \{ 6, 5 \}$  find:

(a)  $X \times (Y \cap Z)$

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(b)  $(X - Y) \times Z$

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(c)  $(X - Y) \times (Y - Z)$

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(4) Identify the following points on a perpendicular net the Cartesian product  $R \times R$ :

A (4,5) , B (6, -3) , C (-2 ,7) , D (-1 , 6) , E (-4, -5) , M (0, 6) , K (9,0).

Then mention the quadrant that each point is located on the perpendicular graphical net. Or the axis it belongs to.

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## Representation of Cartesian Product:

### Sheet No. (3)

[1] If  $X = \{2, -1\}$ ,  $Y = \{4, 0\}$ ,  $Z = \{4, 5, -2\}$

**Find:**

(a)  $X \times Y$

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(d)  $n(X \times Z)$

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(b)  $Y \times Z$

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(e)  $n(Y^2)$

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(c)  $X^2$

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(f)  $n(Z^2)$

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[2] If  $X = \{1, 2\}$ ,  $Y = \{3, 4, 5\}$ , find:

(a)  $X \times Y$

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(b) Represent it by the arrow diagram.

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(c) Represent it by the Cartesian diagram.

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[3] If  $X = \{3, 4, 8\}$ , then find  $X \times X$  and

Represent it with an arrow diagram.

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[4] Draw a perpendicular square net of the Cartesian product  $R \times R$ , then tell the quadrant or the axis where each of the following points is located:

A (3 , 3) , B (3 , -2) , C (-4 , -2) , D (-4 , 3) , E (0 , -3) , F (2 , 0)

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[5] If  $X = [-2, 3]$  find the location which represents  $X \times X$ .

Show which of the following points belongs to the Cartesian product of  $X \times X$ , A (1, 2), B (3, -1), C (-1, 4), D (-2, 0)

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# Representation of Cartesian Product

## Sheet (4)

### **First: Complete:**

- (1) If the point  $(a + 2, 3)$  is located on the Y - axis then  $a = \dots\dots\dots$
- (2) If the point  $(x, x+2)$  is located in the second quadrant, and  $x \in \mathbb{Z}$ ,  
then  $x \dots\dots\dots$
- (3) If  $(1, 2) \in \{1, 6\} \times \{3, X\}$ , then  $X = \dots\dots\dots$
- (4) If  $n(X) = 2$ ,  $n(X \times Y) = 6$ , then  $n(Y) = \dots\dots\dots$
- (5) If  $X \times Y = \{(1, 3), (1, 5), (2, 3), (2, 5)\}$ , then  $X = \dots\dots\dots$ ,  
 $Y = \dots\dots\dots$
- (6) If  $n(Y^2) = 16$ , then  $n(Y) = \dots\dots\dots$
- (7) If  $(3, Y + 2) = (X + 1, 4)$ , then  $X^2 + Y^2 = \dots\dots\dots$   
 $\dots\dots\dots$
- (8) If  $(a^4, 9) = (16, \sqrt{b})$ , then  $a = \dots\dots\dots$   $b = \dots\dots\dots$

### **Second:** If $X = \{1, 2, 3\}$ , $Y = \{4, 5\}$ find

(a)  $X \times Y$

$\dots\dots\dots$   
 $\dots\dots\dots$

(b) represent it by the arrow diagram.

$\dots\dots\dots$   
 $\dots\dots\dots$

(c) represent it by the Cartesian diagram.

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**Third:** If  $A = \{1, 2\}$ ,  $B = \{2, 3\}$ ,  $C = \{3, 4\}$

(a) represent the sets of A, B, C with Venn diagram.

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(b) find  $A \times B$

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(c) find  $A \times (B \cap C)$

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(d) find  $(A - B) \times C$

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(e) find  $C^2$

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(f) find  $(B - C) \times (A \cup B)$

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**Fourth: Complete:**

(1)  $X \times Y = Y \times X$  when ..... = .....

(2) If  $(a, b) \in X \times Y$ , then  $a \in$  .....,  $b \in$  .....

(3)  $X \times Y = \{(a, b): a \in \text{.....}, b \in \text{.....}\}$

## Relations

### Sheet (5)

[1] If  $X = \{-1, 1, 2\}$ ,  $Y = \{2, 4, 6, 8\}$ , and  $R$  is a Relation from  $X$  to  $Y$  where a

$R$  means:  $((b = 2a + 4))$ , for each  $a \in X, b \in Y$ .

Write and represent  $R$  once in an arrow diagram and another by a Cartesian diagram.

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### [2] Complete:

(a) If  $R$  is a relation from  $X$  to  $Y$ , then  $R \subset$  .....

(b) If  $R$  is a relation from a set  $X$  to  $X$ , then  $R$  is called a relation on ....., and  $R \subset$  .....

[3] If  $X = \{-2, -1, 0, 1, 2\}$  and  $R$  is a given relation on  $X$ , where a  $R$

$b$  means: "The number  $a$  is the additive inverse of the number  $b$ "

for each  $a, b \in X$ , write the relation  $R$  and represent it by an arrow diagram and also by, Cartesian diagram.

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[4] If  $X = \{1, 2, 3\}$ ,  $Y = \{12, 21, 47, 52\}$ , and  $R$  is the relation from  $x$  to  $y$  where  $a R b$  means: ( $a$  is a digit from the digits of  $b$ ), for each  $a \in X, b \in Y$

**First:** write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram.

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**Second:** Show which of the following Relations are correct and why?

$1 R 52$  ,  $2 R 21$  ,  $3 R 47$

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## Relations

### Sheet (6)

[1] If  $X = \{1, 2, 4, 6, 10\}$ , and  $R$  is a relation on  $X$ , where  $a R b$  means ( $a$  is a multiple of  $b$ ) for each of  $a, b \in X$  write  $R$  and represent it by an arrow diagram, and also by a Cartesian diagram.

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[2] If  $X = \{2, 4, 5, 7\}$ ,  $Y = \{4, 5, 6, 7, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means ( $a \leq b$ ) for each of  $a \in X$  and  $b \in Y$  write  $R$  and represent it by an arrow diagram and also, by a Cartesian diagram.

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[3] If  $X = \{1, 2, 3\}$ ,  $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$  and  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means "The number  $a$  is the multiplicative inverse of the number  $b$ " for each of  $a \in X, b \in Y$ , write  $R$  and represent it by an arrow diagram and also, by a Cartesian diagram.

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[4] If  $X = \{1, 3, 4, 5\}$ ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means " $a + b = 7$ " for each  $a \in X, b \in Y$  write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram.

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[5] If  $X = \{-1, 0, 1, 2, 3\}$ ,  $Y = \{0, 1, 4, 6, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means " $a^2 = b$ " for each  $a \in X, b \in Y$  write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram.

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[6] If  $X = \{-2, -1, 1, 2\}$ ,  $Y = \{\frac{1}{8}, \frac{1}{3}, 1, 3, 8\}$  and  $R$  is the relation from  $X$  to  $Y$  where  $a R b$  means " $a^3 = b$ " for each  $a \in X, b \in Y$  write  $R$  and represent it by an arrow diagram and also Cartesian diagram.

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[7] If  $X = \{2, 3, 4\}$ ,  $Y = \{6, 8, 10, 11, 15\}$  and  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means "a divides b" for each  $a \in X, b \in Y$ , write the relation  $R$ .

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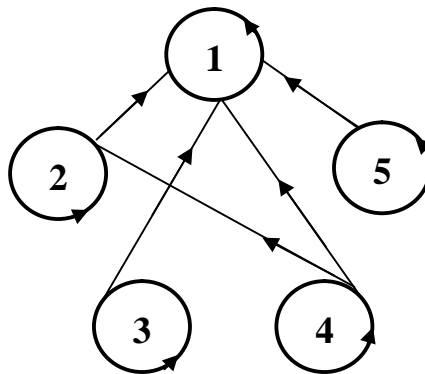
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**[8] The opposite figure:**

Represents the arrow diagram of the given relation  $R$  on the set  $X = \{1, 2, 3, 4, 5\}$  write the relation  $R$  and represent it by a Cartesian diagram.



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## Functions (Mapping)

### Sheet (7)

#### **[1] Complete:**

(a) A relation from  $X$  to  $Y$  is said to be a function if .....

(b)  $f: X \rightarrow Y$  is read as .....

or .....

and we can write it as .....

#### **[2] Put (✓) or (x):**

(a) each function is a relation. ( )

(b) each relation is a function. ( )

(c)  $f: X \rightarrow Y$  can be written as  $f(x)=y$  ( )

(d)  $f: X \rightarrow X$  it's called a function on  $X$

or a function from  $X$  to itself ( )

[3] If  $f$  is a function on  $X$  where :  $X = \{3, 4, 5, 6\}$  and

$f(3) = 3, f(4) = 5, f(5) = 4, f(6) = 5$

**Represent  $f$  by an arrow diagram and also, by a Cartesian diagram.**

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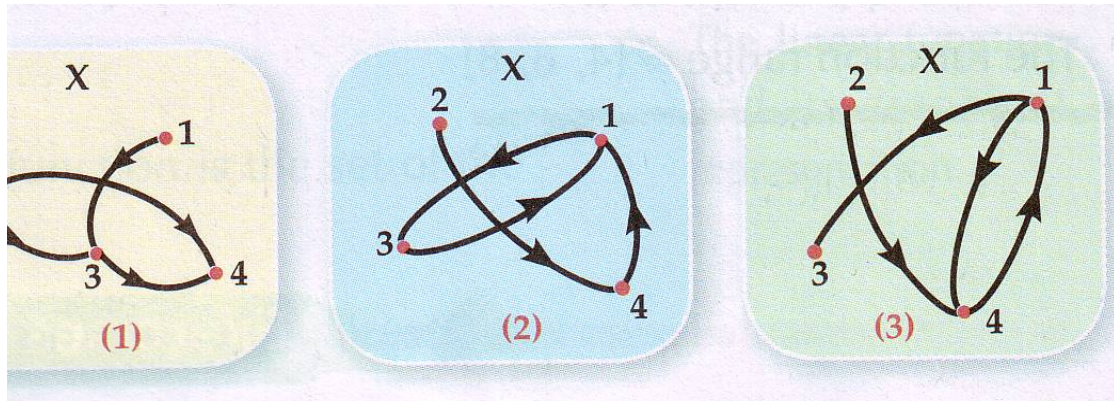
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[4] If  $X = \{1, 2, 3, 4\}$  which of the following arrow diagrams represent a function on the set  $x$ ? and why ?



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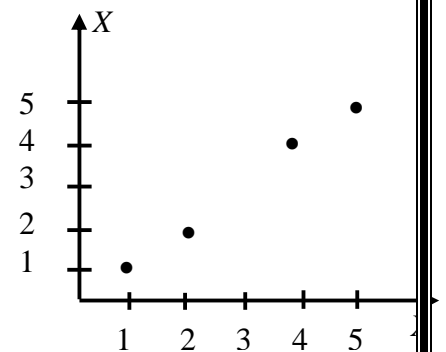
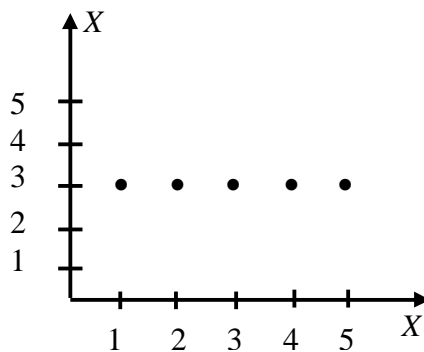
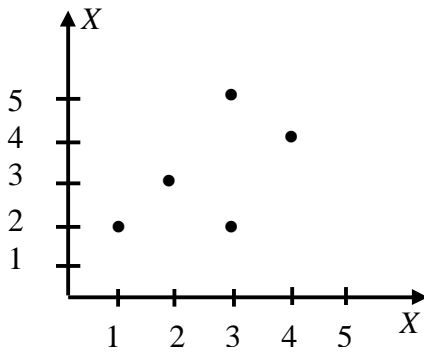
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## Sheet (8)

### Follow functions (mapping)

1) Which of the following Cartesian diagrams represent a function from  $x$  to  $x$  :



2) Complete :

If  $F : x \rightarrow y$  then :

- \*  $X$  is called the ..... of the function  $F$ .
- \*  $Y$  is called the ..... of the function  $F$ .
- \* The set of images of  $x$  is called ..... of the function  $F$ .

3) If  $x = \{-1, 2, 3\}$ ,  $Y = \{2, 3, 5, 7\}$ ,  $F = \{(-1, 3), (3, 5), (2, 7)\}$  then :

**Find** : a) The domain of  $f$  is :

.....

b) The codomain of  $f$  is :

.....

c) The range of f is :

.....

d) Is the range is subset of the codomain .

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4) If  $X = \{2, 3, 4\}$  ,  $Y = \{y : y \in \mathbb{N} , 2 \leq y < 9\}$  , Where  $\mathbb{N}$  is the set of natural number , and  $R$  is a relation from  $x$  to  $y$  where  $a R b$  means

" $a = \frac{1}{2}b$ " for each  $a \in x$  ,  $b \in y$  , write  $R$  and represent it by an arrow

diagram , show that  $R$  is a function from  $X$  to  $Y$  and find its range .

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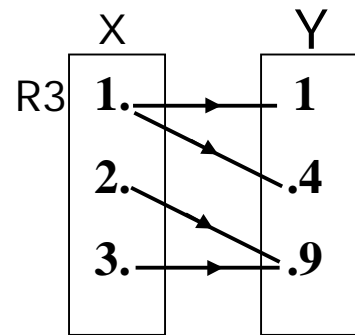
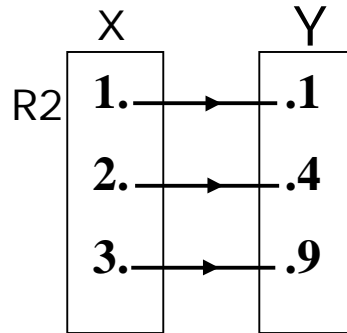
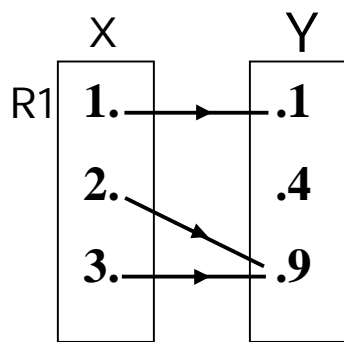
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## Functions ( Mapping )

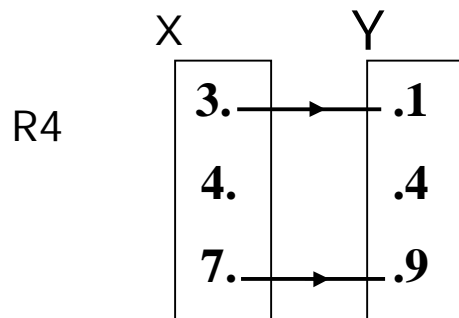
### Sheet (9)

(1) Which of the following relations represent a function from x to y ? If The relation represents a function . then find the function range ?



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2) If  $x = \{2, 5, 8\}$  ,  $y = \{10, 16, 24, 30\}$  and  $R$  is a relation from  $x$  to  $y$  where  $a R b$  means "a is a factor of b" for each  $a \in x$  ,  $b \in y$  Write  $R$  and represent it by an arrow diagram and by Cartesian diagram , Is  $R$  a function? and why ?

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3) If  $X = \{0,1,4,7\}$  ,  $Y = \{1,3,5,6\}$  and  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means "  $a+b < 8$ " for each  $a \in X$  ,  $b \in Y$  , Write  $R$  and represent it by an arrow diagram and also , by a Cartesian diagram. Is  $R$  a function ? and why ?

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4) If  $x = \{1, 2, 4, 6, 10\}$  and  $R$  is a relation on  $X$  where  $a R b$  means : "a is a multiple of b" for each of  $a, b \in x$ , write  $R$  and represent it by an arrow diagram and also , by a Cartesian diagram. Is  $R$  a function ? and why ?

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5) If  $x = \{1, 2, 3, 6, 11\}$  and  $R$  is a relation on  $X$  where  $a R b$  means: " $a+2b$ " = an odd number for each of  $a, b \in x$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function ? and why ?

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6) If  $x = \{3, 4, 5\}$  and  $y = \{5, 6, 8, 10\}$  and  $R$  is a relation from  $x$  to  $y$  where  $aRb$  means " $a = \frac{1}{2}b$ " for each  $a \in x$  and  $b \in y$ . Write  $R$  and show that  $R$  is a function and Write its range .

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7) If  $x = \{2, 3, 5, 6\}$  and  $R$  is a relation on  $x$  where  $a R b$  means " $a + b \leq 9$ " for each  $a, b \in x$ . Write  $R$  and represent it with an arrow diagram and a Cartesian diagram . Is  $R$  a function or not ? explain why ?

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8) If  $x = \{-2, 2, 5\}$ ,  $y = \{3, 7, L\}$ ,  $R$  is a relation from  $X$  to  $Y$  where  $a R b$  means " $b = a^2 - 1$ " for each  $a \in x$ ,  $b \in y$ .

(i) Find the value of  $L$  .

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(ii) Represent the relation by an arrow diagram and mapping .

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## Polynomial functions [ linear function ]

### Sheet (10)

#### 1) Complete :

A) The function F where  $f(x) = 6x^7 + 2x^5 - 4x + 1$  is a polynomial of ..... degree .

B) The degree of the polynomial is the ..... of the variable in the function rule .

C) If  $f(x)$  is a polynomial of  $n$ -degree ,

then :  $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$  , Where

$a_0, a_1, a_2, a_3, \dots, a_n \in \dots$  ,  $n \in \dots$  ,

$a_n \neq \dots$

D) If  $F:R \rightarrow R$  , then the domain is ..... and the co-domain is .....

#### 2) Which of the following functions represents polynomial :-

a)  $f_1(x) = X^3 + X^2 + 3$

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b)  $f_2(x) = X^3 + \frac{1}{X} + 7$

.....

c)  $f_3(x) = X^2 + \sqrt{X} + 8$

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d)  $f_4(x) = X(X + \frac{1}{X} - 2)$

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3) If  $f: \mathbb{R} \rightarrow \mathbb{R}$  then mention the degree of the function in the following :-

a)  $f(X) = 3 - 2X$

.....

b)  $f(X) = X^2 - (X^2 - 3)$

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c)  $f(X) = X(X - 2X^2)$

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d)  $f(X) = X^2(X - 3)^2$

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4) If  $f(x) = x^2 - x + 3$  Then Find :  $f(-2)$  ,  $f(0)$  ,  $f(\sqrt{3})$

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5) If  $F(x) = x^2 - 3x$  ,  $g(x) = x - 3$

A- Find  $F(\sqrt{2}) + 3g(\sqrt{2})$

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B- Prove that  $F(3) = g(3) = 0$

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6) If  $f: \mathbb{R} \rightarrow \mathbb{R}$  Where  $f(X) = ax + b$ ,  $a, b \in \mathbb{R}$ ,  $a \neq 0$

a- Find the degree of f

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b- Complete :  $f(x)$  is called .....function or a function  
 of ..... and represented by a .....

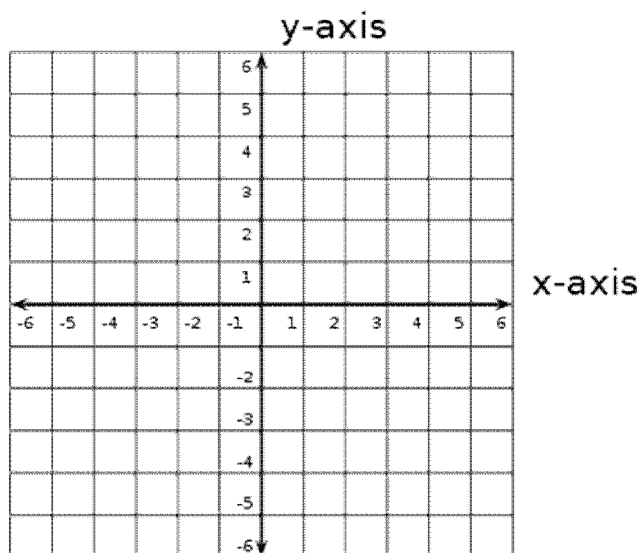
c- give two examples for a linear function .

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7) Represent graphically the function

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x - 3$$

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8) If  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = ax$ , Where  $a \neq 0$ , then this Function is  
 represented by ..... passing through.....

9) **Complete** :

a-  $f(x) = 5$  is represented graphically by a straight line parallel to ..... and cut ..... axis at the point ( ..... , ..... ) and apart ..... unit from ..... axis in the ..... direction .

b-  $f(x) = -4$  is represented graphically by a ..... parallel to ..... axis and cut ..... axis at ..... and apart ..... units from ..... axis at the ..... direction .

c-  $f(x) = C$  , where  $C \in \mathbb{R}$  , is called ..... function .

d-  $f(x) = a x + b$  ,  $a, b \in \mathbb{R}$  ,  $a \neq 0$  is called ..... function.

e- The linear function given by the rule  $Y = 4x + 2$  is represented graphically by a straight line intersecting the x- axis at ( ..... , ..... ) and intersecting the y-axis at ( ..... , ..... )

f- If the point  $(2,a)$  is located on the straight line which represents the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = 2x-3$  , then  $a =$  .....



g- The linear function given by the rule  $y=3x-5$  is represented graphically by a straight line cut x-axis at the point (..... ,.....) and cut y-axis at the point (..... , ..... ).

h- If the point ( a , -1 ) is located on the straight line which represents the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  Where  $f(x) = 7x + 6$  , then a =  
.....

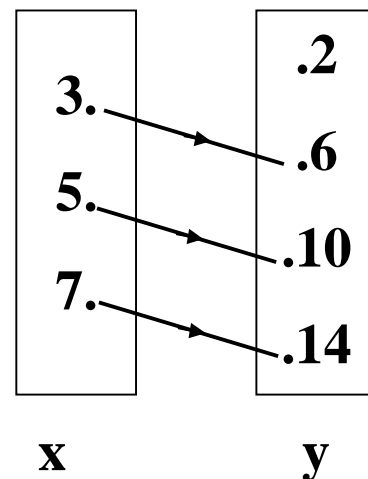
## Linear and constant functions

### Sheet (11)

#### (1) Complete:

- a) The linear function given by the rule  $y = 2x - 1$  is represented graphically by a straight line intersecting the y-axis at the point ( ..... , ..... )
- b) The linear function given by the rule  $y = 3x + 6$  is presented graphically by a straight line intersecting the x-axis at the point .....
- c) If the point  $(a, 3)$  is located on the straight line which represents the function  $F: \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = 4x - 5$ , then  $a$  equals.....
- d) If  $x = \{ 2, 4, 6 \}$ ,  $y = \{ 3, 5, 7, 9 \}$  then  $n(x \times y) = \dots\dots\dots$
- e) If the point  $(x, x^2 - 4)$  lies in the second quadrant of the grid square  $\mathbb{R} \times \mathbb{R}$  then  $x$  may be equals .....  $(\sqrt{3}, -\sqrt{5}, -\sqrt{3}, \sqrt{5})$

- f) The figure opposite represent the relation  $R$  from  $x$  to  $y$ , if  $a R b$  for each  $a \in x$ ,  $b \in y$ , then  $a : b = \dots\dots\dots$



- g) If  $x = \{ 3, 5, 7 \}$ ,  $n(y) = 4$  and the function  $f: x \rightarrow y$ ,  $f(x) = 2x - 5$ , then  $y$  may be equal .....

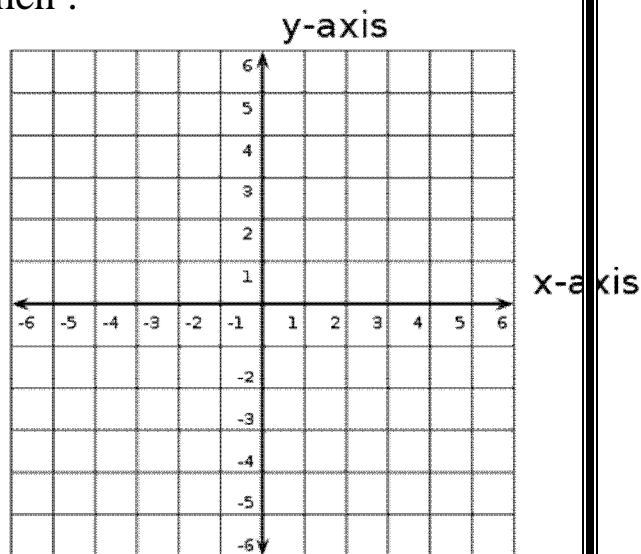
2) If  $x = \{3, 4, 5\}$ ,  $n(y) = 5$  and the function  $f: x \rightarrow y$ ,  $F(x) = 3x-1$  then Find  $y$  and represent the function by an arrow diagram .

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3) The function  $F: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 3x-1$ , then :

a- represent the function graphically .

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b- If  $f(k) = 29$ , then what is the value of  $k$  .

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c-Find the points of the intersection of the function with the axes of coordinates.

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4) If  $f: \mathbb{R} \rightarrow \mathbb{R}$ , mention the degree of  $f$ ; then **Find** :  $f(-2)$ ,  $f(0)$ ,  $f(\frac{1}{2})$

When : a)  $f(x) = 3$

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b)  $f(x) = 3 - 2x$

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c)  $f(x) = x^2 - 4$

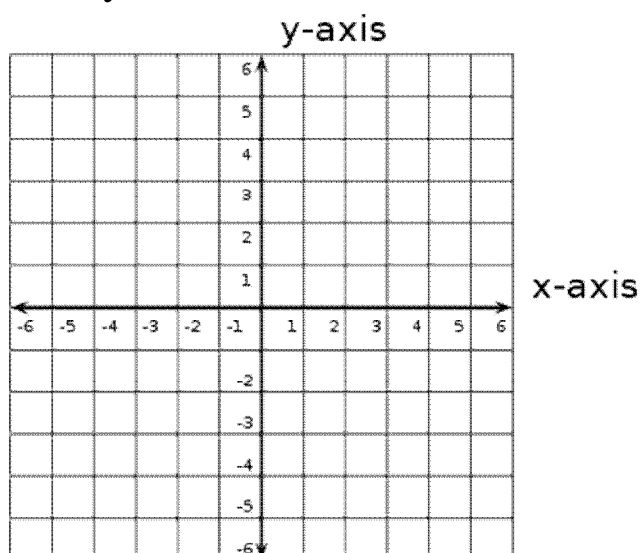
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5) Represent graphically the following linear functions and find the points of intersection of the straight line by the two coordinate axes

:

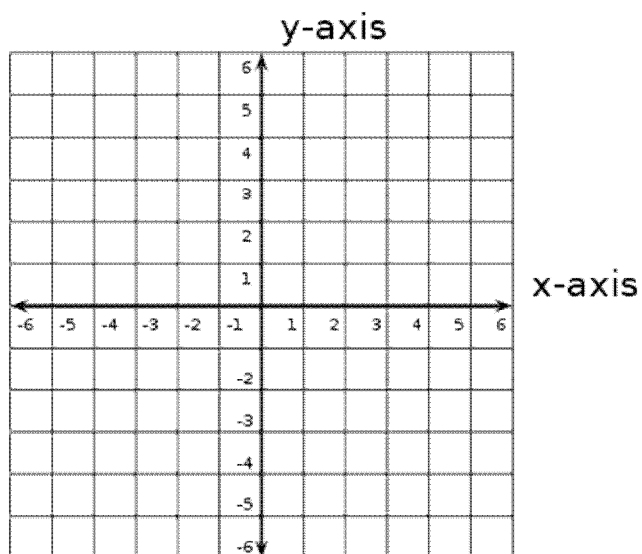
a-  $f(x) = 2x$

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b-  $f(x) = -\frac{1}{2}x$

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c-  $f(x) = 2x+1$

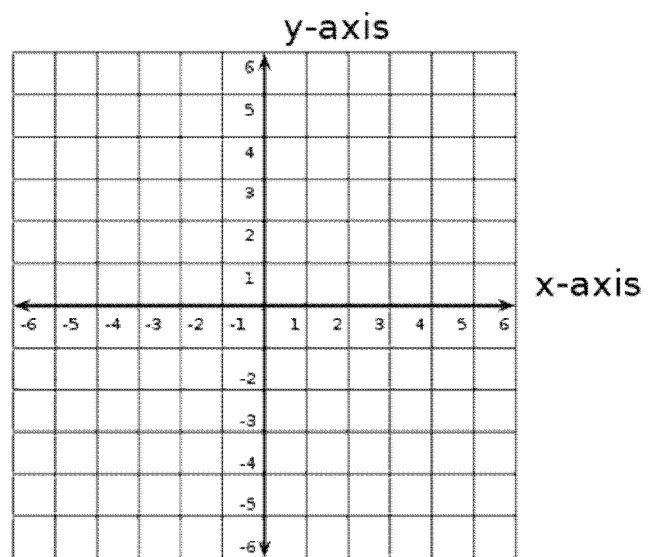
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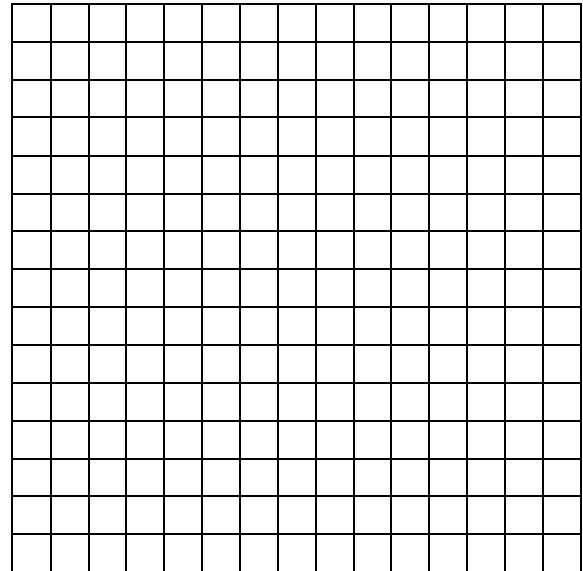
## Quadratic function

### Sheet (12)

1) Represent graphically the quadratic function F.

Where  $F(x) = x^2$ ,  $x \in \mathbb{R}$  Consider  $x \in [-3, 3]$ .

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2) In the pervious example :

a) Draw the axis of symmetry to the curve .

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b) Find the equation of the axis of symmetry .

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c) Find the coordinate of the vertex of the curve .

.....

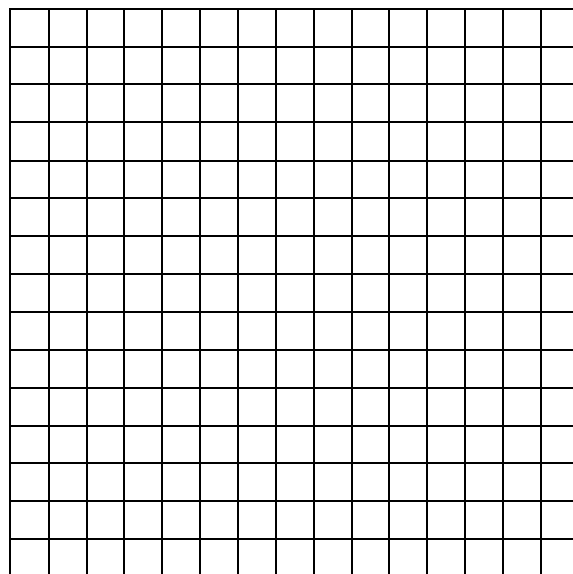
d) The minimum or maximum value of the function.

.....



3) Represent graphically the quadratic function F. Where :  $f(x) = -x^2$ ,  
 $x \in \mathbb{R}$ , where  $x \in [-3, 3]$ .

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4) **In the previous drawing find :**

a) The axis of symmetry .

.....

b) The equation of the axis of symmetry .

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c) The coordinate of the vertex.

.....

d) The maximum or minimum value of  $f(x)$ .

.....

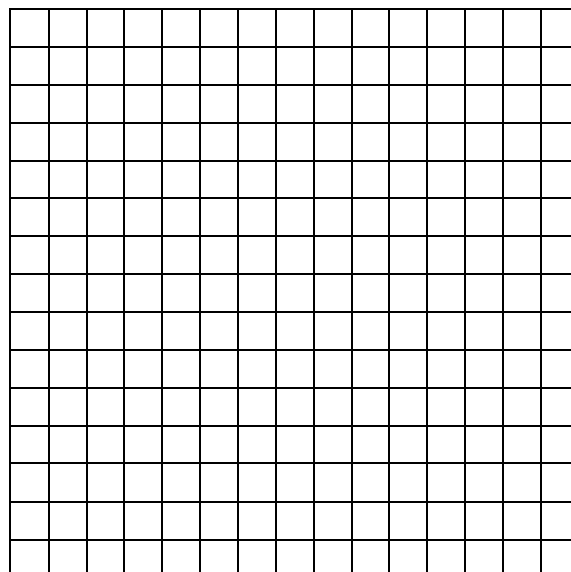
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5) Represent graphically each of the following function and from the drawing deduce the coordinate of the vertex of the curve, and the equation of the symmetry axis and the minimum value of the function:

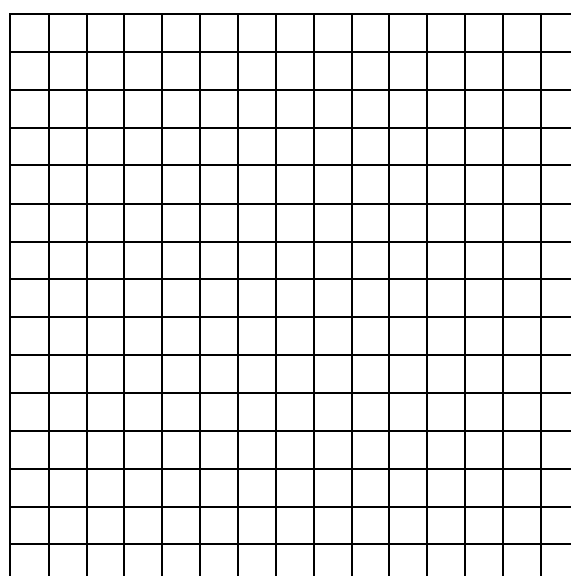
a)  $f(x) = x^2 - 2$  where  $x \in [-3, 3]$

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b)  $f(x) = (x-2)^2$  where  $x \in [-1, 5]$

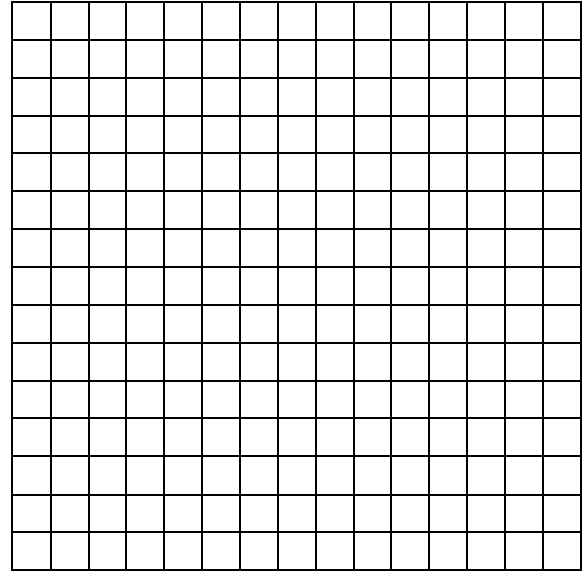
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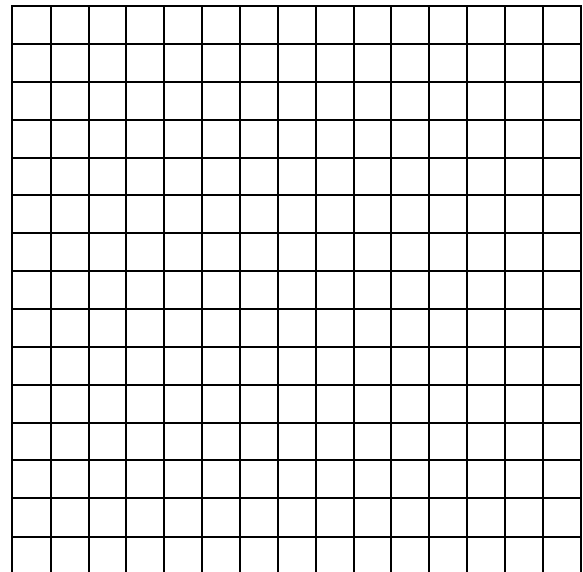
c)  $f(x) = x^2 + 2x + 1$  where  $x \in [-4, 2]$

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d)  $f(x) = 2 - x^2$  where  $x \in [-3, 3]$

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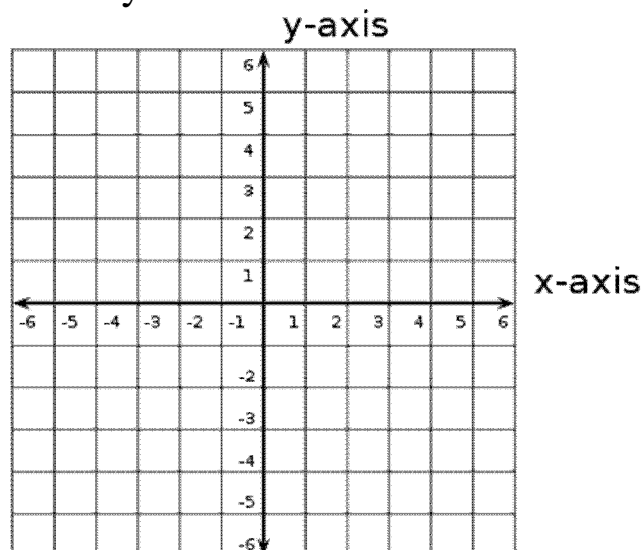
# Quadratic function

## Sheet(13)

1) Represent graphically the following linear functions and find the points of intersection of the straight line by the two coordinate axes

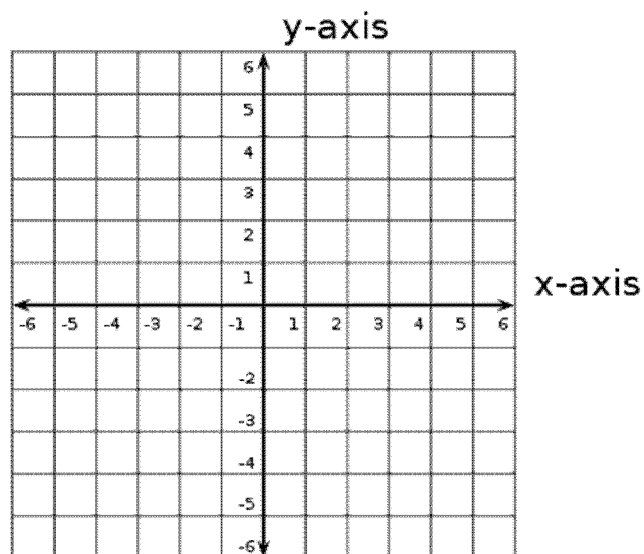
a)  $f(x) = 2 - x$

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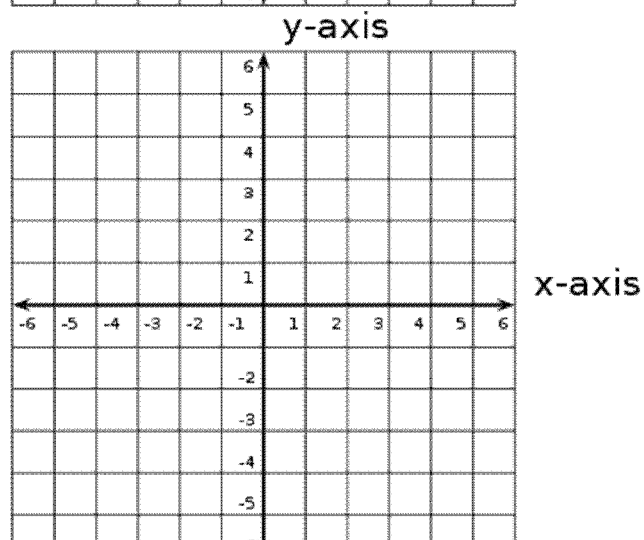
b)  $f(x) = 3x - 1$

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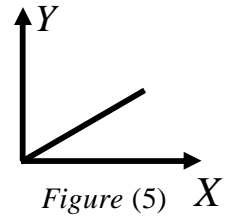
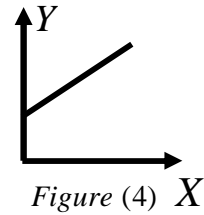
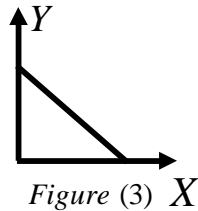
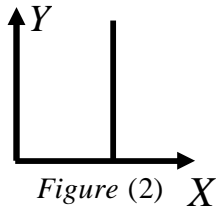
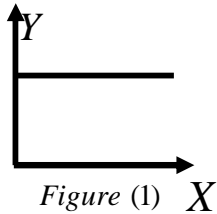


c)  $f(x) = -2x + 3$

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2) A pavement company gets paid 100.000 pounds ( fixed fee) then 30 pounds for each meter , If  $x$  ( The length of the paved road in meters ) and  $y$  is ( The total cost that the company receives ) .



a) The figure that represents the relation between  $x$  and  $y$  is the figure number .....

b) Which of the following relations represents the previous information :

(a)  $y = 30 x$

(b)  $y = 30 x + 100000$

(c)  $y = 100000 x + 30$

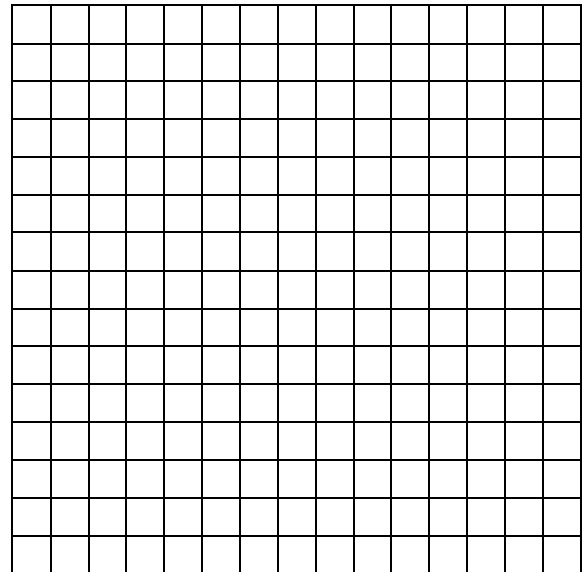
(d)  $y = 3000000 x$

3) Represent graphically each of the following functions :

Find the axis of symmetry , the equation of the axis of symmetry , the vertex point and the minimum or maximum value for the pervious functions given in a and b .

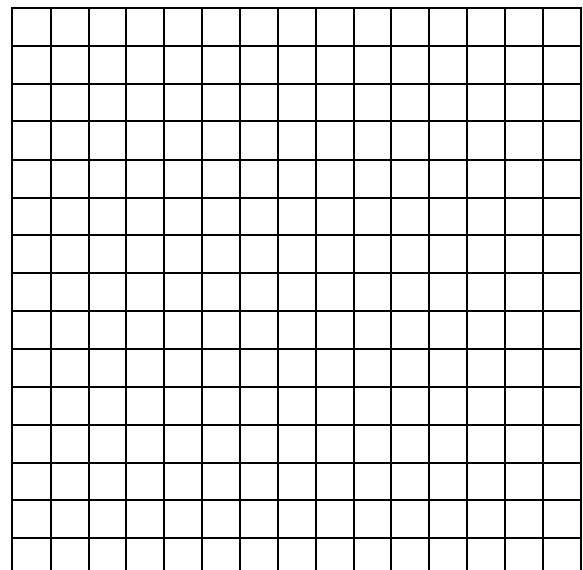
a)  $f(x) = x^2 - 3$  where  $x \in [-3, 3]$

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b)  $f(x) = 1 - 3x + x^2$  where  $x \in [-1, 4]$

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c)  $f(x) = -2x$

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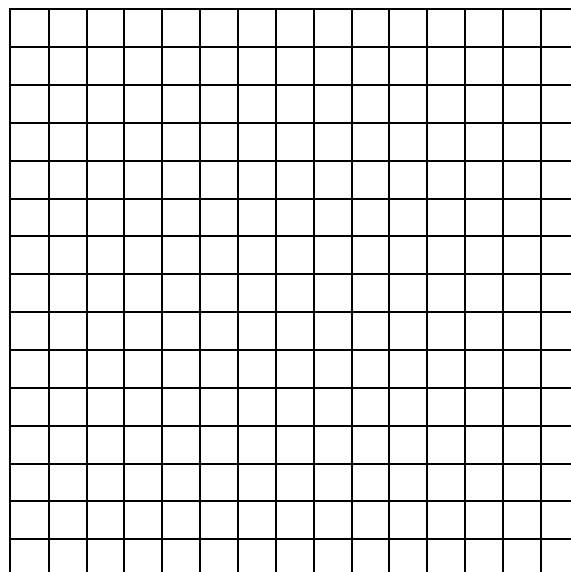
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## Revision sheet on unit (1)

### Sheet (14)

1) If  $X = \{0, 1, 4, 7\}$ ,  $Y = \{1, 3, 5, 6\}$ ,  $R$  is a relation from  $X$  to  $Y$

Where  $a R b$  means : “  $a + b < 6$ ” For each  $a \in x$ ,  $b \in y$ , Write  $R$  and represent it by an arrow diagram and by Cartesian diagram .

Is  $R$  a function ? tell the reason .

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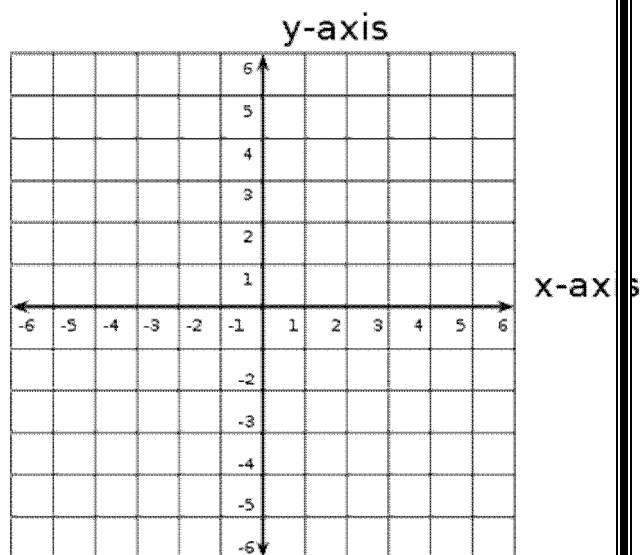
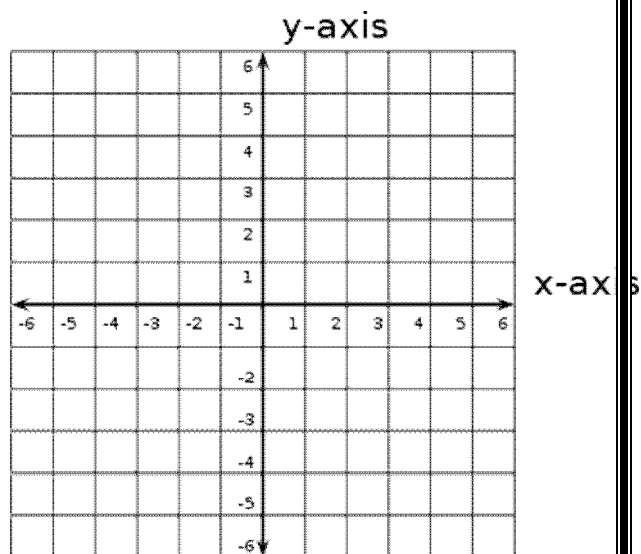
2) Represent graphically each of the following function :

a)  $F(X) = 3X - 1$

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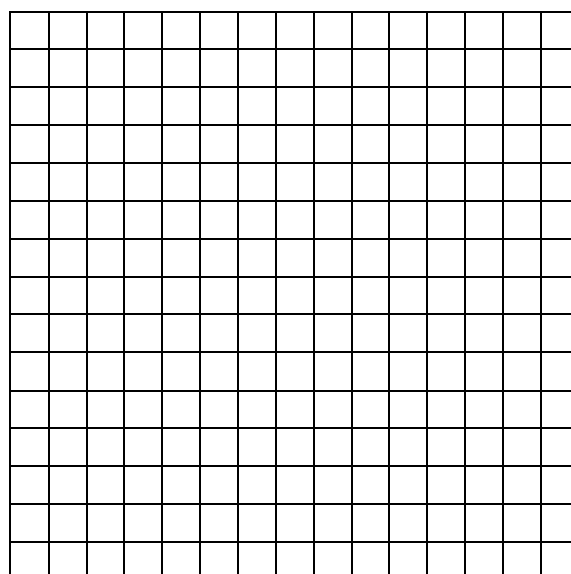
b)  $F(X) = -3X$

.....  
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3) While Karim was reading a book, he found that after 3 hours 50 pages remained, and after 6 hours, 20 pages remained. If the relation between time (t) and the number of pages (b) is a linear function .

a) Represent graphically the relation between t and b then find the algebraic relation between the two variables.



b) What is the time that should be taken to finish the book ?

.....

c) What are the number of pages remaining when Karim began to read ?

.....

4) The opposite figure : Represents the curve of the function f:

Where :  $f(x) = m - x^2$  , If  $ao = 4$  units .

Find :

a) The value of m .

.....

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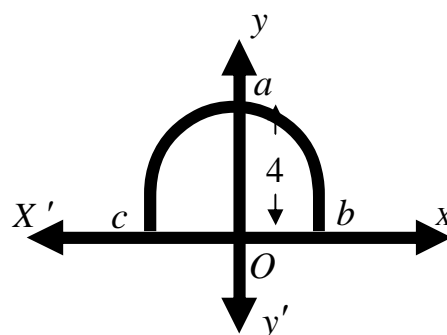
b) The coordinates of b and c .

.....

c) The area of the triangle with vertices a , b and c .

.....

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5) Graphically represent the quadratic function F Where

$f(x) = x^2 - 6x + 7$  ,  $x \in \mathbb{Z}$  , Taking  $x \in [0 , 6]$  and from the

drawing , Find :

a) The vertex of the curve .

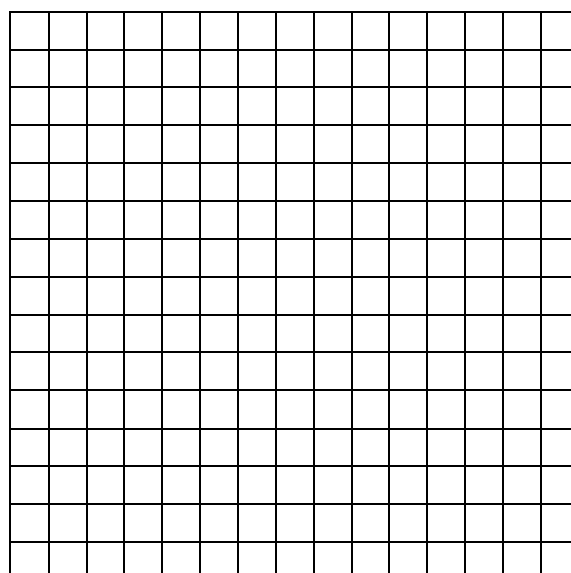
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b) The minimum value of the function f .

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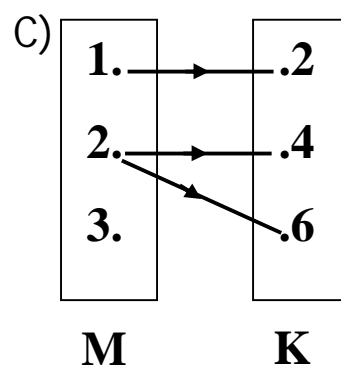
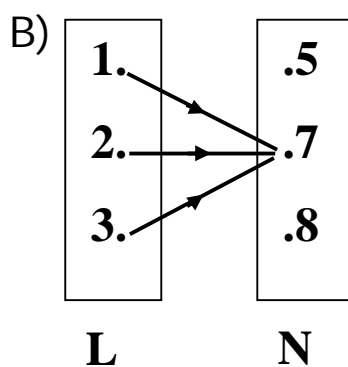
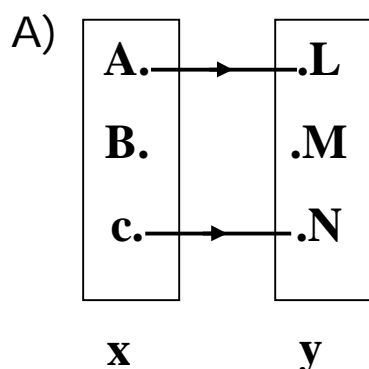
c) The S.S. of the equation  $f(x) = 0$

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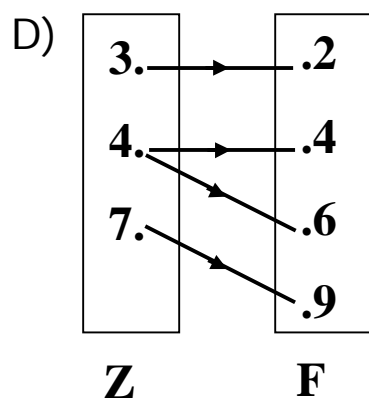




6) Which of the following relations represent a function :



.....



.....

7) If  $x = \{ 2, 4, 6 \}$  and the function  $f: x \rightarrow \mathbb{R}, f(x) = 2x + 3$ , then find the range of the function  $f$

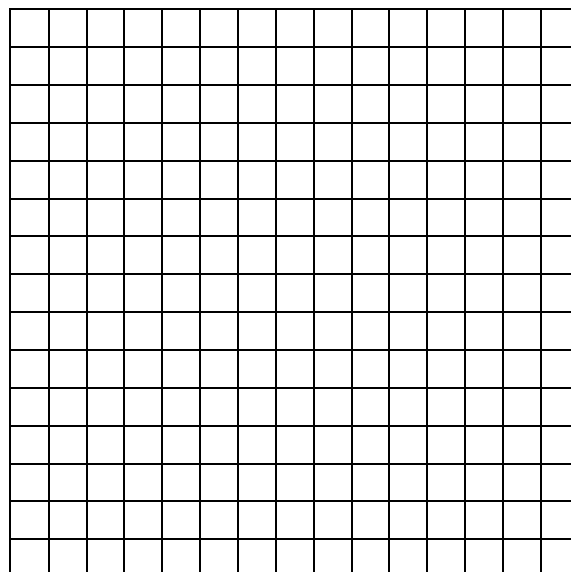
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8) Graph the function  $f(x) = x^2 - 4x - 3$  on the interval  $[-2, 6]$ , then find the solution set of the equation  $f(x) = 0$

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9) A bomb was throw on from a canon in path follow the relation  $y = 12 - x - x^2$ ; where  $x$  is the horizontal distance by km,  $y$  is the height of the bomb from the earth by km. What is the horizontal distance from the canon that the bomb extend ?

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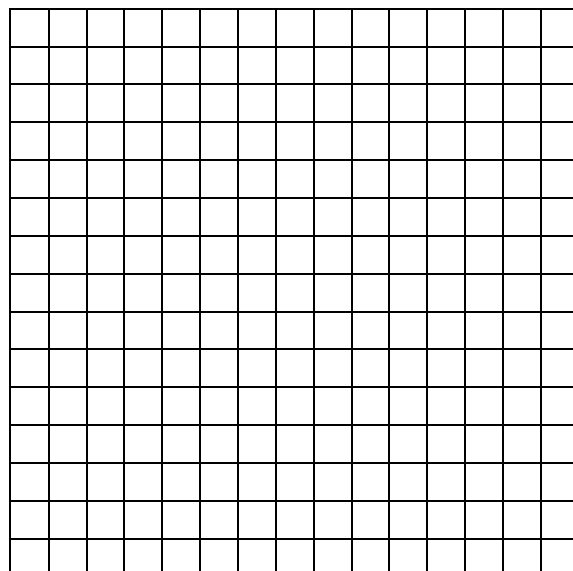
10) If  $x = \{2, 4, 6\}$ ,  $n(y) = 4$  and the function  $f: x \rightarrow y$ ,  $f(x) = x^2 - 1$ , then which of the following sets can be equal to  $Y$ .

- a)  $\{3, 7, 13\}$                       b)  $\{3, 15, 25, 45\}$
- c)  $\{3, 15, 35\}$                      d)  $\{3, 15, 25, 35\}$

.....

11) Graph the function  $f(x) = 9 - x^2$  at the interval  $[-3, 3]$  and find the maximum value of the function , find also the axis of symmetry and it's equation also find the vertex point .

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12) If  $x = \{4, 6, 8, k\}$  ,  $y = \{2, 3, 4, 5\}$  ,  $R$  is a relation from  $x$  to  $y$  where  $a R b$  means “  $b = \frac{1}{2} a$  ” for each  $a \in x$  ,  $b \in y$  :

a) Find the value of  $K$ .

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b) Represent  $R$  by mapping diagram .

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# Algebra

## Unit (2)

**Ex. (2-1) P. Book**

**1- Find the number which if added to the terms of the ratio 7 : 11 it will be 2 : 3**

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**2- find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 it becomes 3 : 5**

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**3- Two integer numbers, the ratio between them is 3 : 7 and if subtracted 5 from each term, the ratio between each of them becomes 1 : 3 Find the two numbers?**

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**4- Two integer numbers, the ratio between them is 2 : 3 if you add to the first 7 and subtract from the second 12, the ratio between them becomes 5 : 3 find the two numbers?**

.....

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**5- Find the number that if subtracted thrice from one of the two terms of ratio  $\frac{49}{69}$  the ratio becomes  $\frac{2}{3}$ .**

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**6- Find the number which if its square is added to each of the two terms of ratio 7 : 11 it becomes 4 : 5.**

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## Proportion

1- If  $\frac{x}{y} = \frac{2}{3}$  , Find the value of :  $\frac{3x + 2y}{6y - x}$

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2- Find the fourth proportional for the numbers 4 , 12 , 16.

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3- Find the number that if added to the numbers 3 , 5 , 8 , 12 it becomes proportional.

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**4-a) Find the second proportional number of the numbers**

**2,...,4,6**

.....

.....

**b) Find the third proportional number of the numbers 8,6 ,...,12**

.....

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**5) If  $\frac{a}{b} = \frac{3}{5}$  find the value of  $7a + 9b : 4a + 2b$**

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**6) If :  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$  , then prove that :  $2a - 5b + 3c =$  one of these ratios.**

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7) If : a, b, c and d are proportional quantities ,then prove that

$$\frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$$

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8) If  $\frac{a}{b} = \frac{c}{d}$  , then prove that :

a)  $\frac{a + b}{b} = \frac{c + d}{d}$

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b)  $\frac{a - b}{b} = \frac{c - d}{d}$

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**Ex. (2-2) P. Book**

**If :  $\frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$  prove that each ratio is equal to 2**

**(unless :  $x + y = 0$ ) then find the ratio  $x : y : z$**

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**2- If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$  then find the value of  $x$**

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**3- If  $a : b : c = 5 : 7 : 3$  and  $a + b = 27.6$   
then find the value of  $a$ ,  $b$  and  $c$**

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**4- If  $x, y, z$  and  $L$  are proportional quantities then prove that :**

**A)  $\left(\frac{x + y}{z + L}\right)^2 = \frac{2x^2 - 3y^2}{2z^2 - 3L^2}$**

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**B)  $\sqrt[3]{\frac{5x^3 - 3z^3}{5y^3 - 3L^3}} = \frac{x + z}{y + L}$**

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5- If  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$  then prove that:

A)  $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

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B)  $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

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6- If a , b , c and d are four real proportional quantities then prove that:

A)  $\frac{ac}{bd} = \left( \frac{a - c}{b - d} \right)^2$

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$$\text{B) } \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a + c}{b + d}$$

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**7- If b is the middle proportional between a and c, then prove that:**

$$\text{A) } \frac{a+b+c}{a^{-1} + b^{-1} + c^{-1}} = b^2$$

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$$\text{B) } \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$$

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**8 - If a , b , c and d are in continued proportional ,  
then prove that:**

**A)**  $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$

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**B)**  $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

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**C)**  $\frac{a}{b + d} = \frac{c^3}{c^2d + d^3}$

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**D)**  $\frac{c^2 - d^2}{a - c} = \frac{bd}{a}$

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**9- If 5a , 6b , 7c and 8d are positive quantities in continued proportional.**

**Prove that:**

$$\sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a + 6b}{7c + 8d}}$$

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**Ex. (2-3) P. Book**

(1) If  $Y \propto X$  then  $Y = 14$  when  $X = 42$  , then find :

a- The relation between X and Y

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b- Find the value of Y when  $X = 60$

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(2) If  $Y \propto \frac{1}{X}$  and  $Y = 3$  when  $X = 2$  :

a- Find the relation between X and Y.

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b- Find the value of Y when  $X = 1.5$

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**(3) If the relation between velocity (v) in (m/sec) and time t (sec) is**  
 **$V = 9.8 t$**

**a) Determine the kind of variation between v and t**

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**b) i) Find the values of v when  $t = 2$  seconds,  $t = 4$  seconds.**

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**ii) Find the value of t when  $v = 24.5$  m/sec**

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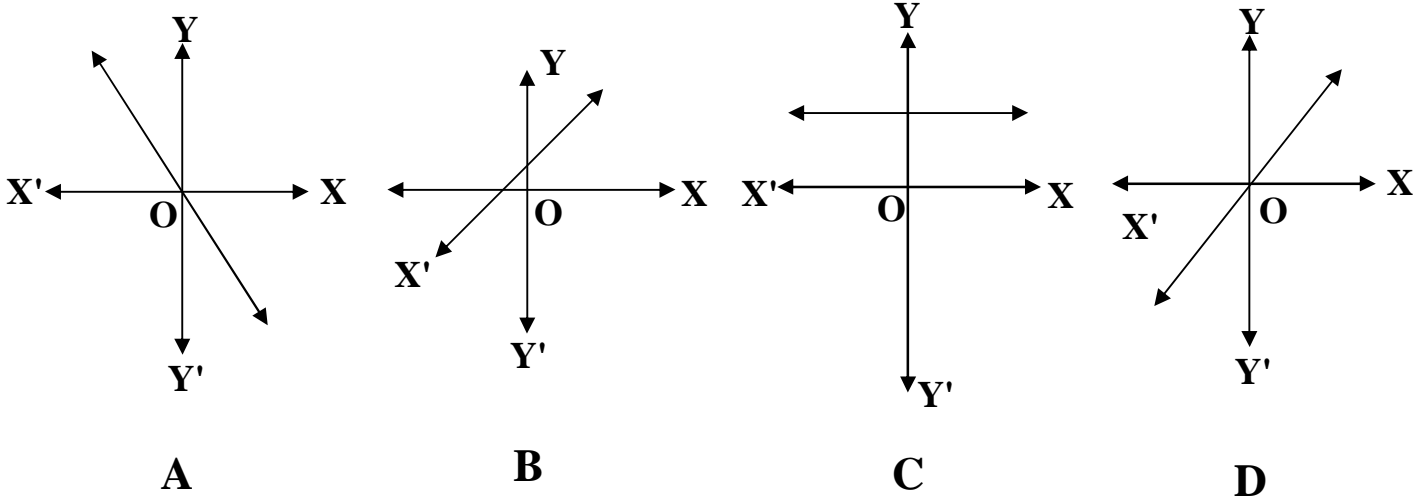
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**(4) If the height of a right constant cylinder (constant volume) is**  
**(h) varies inversely as the square of its radius length r. if the**  
**(h) = 27cm, when the radius = 10.5cm. Find (h) when  $r = 15.75$  cm.**

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**(5) Choose the correct answer from the given answers:**

**1- The graphical form represents the direct variation between X and Y is:**



**2- The relation represents the direct variation between the two variables X and Y which is:**

- A)  $XY = 5$       B)  $Y = X + 3$       C)  $\frac{X}{3} = \frac{4}{Y}$       D)  $\frac{X}{5} = \frac{Y}{2}$**

**3- If Y varies inversely with X , and  $X = \sqrt{3}$  when  $Y = \frac{2}{\sqrt{3}}$  then the constant proportional equals:**

- A)  $\frac{1}{2}$       B)  $\frac{2}{3}$       C) 2      D) 6**

(6) From the data of the following table, answer the following questions:

X	2	4	6
Y	6	3	2

A) Show the kind of variation between Y and X .

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B) Find the constant proportion.

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C) Find the value of Y when  $X = 3$ .

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D) Find the value of X when  $Y = 2\frac{2}{5}$ .

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## General Exercise on unit (2) . p.book

**1 ) If the total cost of a trip is (y) some of it is constant (a) and the other is directly proportional with the number of participant (x) then: choose the correct answer :-**

A)  $y = a x$

B)  $y = a + X$

C)  $y = a + \frac{m}{x}$  ( m constant  $\neq 0$ )

D)  $y = a + m x$  ( m constant  $\neq 0$ )

**2) If  $y \propto x$  and  $y = 40$  When  $x = 14$  , then Find  $x$  when  $y = 80$**

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**3) A car moves with a uniform velocity where the distance varies directly with time. If the car covers 150km in 6 hours, find the distance covered by that car in 10 hours?**

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4) If the weight of a body on the moon ( $w$ ) is directly proportions with its weight on the ground ( $R$ ) if the body weights 84 kg on the ground and its weight on the moon is 14kg. What will be its weight on the moon if its weight on the ground is 144 kg?

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5) If  $Y$  changes inversely with  $x$  and  $y = 2$  When  $X = 4$ .  
Then Find the value of  $Y$  When  $X = 16$

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6) If  $y \propto x$  prove That :       $Y^2 + X^2 \propto Y^2 - X^2$

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7) If  $a, b, c$  and  $d$  are in continued proportional, Then prove that :

A )  $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

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b)  $\frac{2a + 3d}{3a - 4d} = \frac{2a^3 + 3b^3}{3a^3 - 4b^3}$

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8) If  $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$  Then prove that

$$\frac{2x + y}{4a + 4b - c} = \frac{2x + 2y + z}{3a + 6b}$$

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**9)** *X, y, z are Three proportional sides in a triangle and  $X + Y = 15\text{cm}$  ,  
 $y + z = 22.5\text{cm}$ . Find X : Y*

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**10)** *Through the interest of the Egyptian authorities with the villages , a budget of  $1.85 \times 10^6$  pounds was set for one of the villages to build a school , a medical unit and a youth center , if the costs of the school is  $\frac{3}{2}$  of the cost of the medical unit and the cost of the medical unit is  $\frac{5}{6}$  of the costs of the youth center , What is the cost of each of them ?*

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***11) If the needed hours to fulfill a work ( $t$ ) is proportionally inverse with the number of workers ( $x$ ) Who do the work, if 6 workers fulfilled the work in four hours , What is the time needed for 8 workers to fulfill this work ?***

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## General Exercise Based on unit (2)

1) If  $\frac{a + b}{3} = \frac{b + c}{6} = \frac{c + a}{5}$  Then prove that:

$$\frac{a + b + c}{a} = 7$$

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2) If  $y = a - 9$  and  $y \propto \frac{1}{x^2}$  and  $a = 18$  When  $x = \frac{2}{3}$ , Then Find the relation between  $y$  and  $x$ , Then deduce the value of  $y$  when  $x = 1$

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3) If  $\frac{21X - Y}{7X - Z} = \frac{y}{z}$  Then prove that  $y \propto z$

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4) If  $x^4 y^2 - 14x^2 y + 49 = 0$  then prove that  $y \propto \frac{1}{x^2}$

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5) If a weight of a body on Earth (R) directly changes with its weight on the moon (w), If  $R_1=128$  kg,  $w_1=35$ kg then find  $w_2$  and  $R_2=312$  kg.

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6) If the speed of expression V of water to pass through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and  $v = 5$ cm /s when  $r = 3$ cm. Find v When  $r = 2.5$ cm

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## Nov. Revision sheet (Algebra)

- (1) What is the number that must be added to the two terms of the ratio  $5 : 37$  to be equal to the ratio  $\frac{1}{3}$  ?

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- (2) Find two numbers which the ratio between them is  $7 : 12$  and one of them exceeds the other by  $275$ .

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- (3) Complete:

a) 3 pounds : 240 piaster = ..... : ..... (in simplest form)

b) If  $(2x - 3) : (X - 5) = 1 : 4$  , then  $X = \dots\dots\dots$

$$c) \frac{a+b}{a} = \frac{\dots\dots\dots}{a^2}$$

$$d) \frac{a^2c}{b^2d} : \frac{ac^2}{d^2b} = \dots\dots\dots : bc$$

(4) **Complete:**

a) The third proportion of 8 , 6 , 12 is .....

b) The fourth proportion of 4 , 12 , 16 is .....

c) The fourth proportion is .....

d) The proportion is .....

e) If  $5a - 4b = 0$  , then  $\frac{a}{b} = \dots\dots\dots$

f) If  $\frac{a}{2} = \frac{b}{3}$  , then  $\frac{3a}{3b} = \dots\dots\dots$

g) If  $\frac{a+b}{a+2b} = \frac{2}{5}$  , then  $a : b = \dots : \dots$

h) If  $4X^2 + 9Y^2 = 12 XY$  , then  $\frac{X}{Y} = \dots\dots\dots$

i) If  $2X = 7Y$  , then  $\left(\frac{X}{Y}\right)^{-1} = \dots\dots\dots$

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(5) If  $\frac{2x+3}{2x-3} = \frac{2Y+5}{2Y-5}$  , prove that:  $\frac{X}{Y} = \frac{3}{5}$

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(6) If  $\frac{X}{3} = \frac{Y}{4} = \frac{Z}{5}$ , prove that:  $\frac{2Y - Z}{3X - 2Y + Z} = \frac{1}{2}$

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(7) Prove that a , b , c , d are proportional if:  $\frac{a - b}{a + b} = \frac{c - d}{c + d}$

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(8) If  $\frac{X+Y}{3} = \frac{Y+Z}{5} = \frac{Z+X}{6}$ , prove that:  $\frac{X+Y+Z}{2X+3Y+3Z} = \frac{7}{19}$

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(9) If  $\frac{a}{2X+Y} = \frac{b}{3Y-X} = \frac{c}{4X+5Y}$ , prove that:  $\frac{a+2b}{4b+c} = \frac{7}{17}$

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(10) Find the middle proportion between:

a) -2 , -8

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b)  $a^4$  ,  $a^2b^2$

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(11) Find the third proportion between:

a)  $X^2$  , - 5X

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b)  $X^2$  ,  $-3X^2$

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(12) If b is the middle proportion between a and c , prove that:

$$\frac{a-b}{b-c} = \frac{a+3b}{3c+b}$$

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(13) If a , b , c , d are in contained proportion , prove that:

$$\frac{3a+5c}{3b+5d} = \frac{a-4c}{b-4d}$$

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(14) If  $Y^2 = XZ$ , prove that:

$$\frac{X(X - Y)}{Y(Z - Y)} = \frac{Y^2}{Z^2}$$

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(15) If  $\frac{X^2 - Y^2}{Y^2} = \frac{Y^2 - Z^2}{Z^2}$ , prove that:

Y is the middle proportion between X and Z, where XZ is a positive quantity.

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(16) If a varies inversely as b and  $a = 12$  as  $b = 8$  Find:

(i) The value of a as  $b = 1.5$

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(ii) The value of b as  $a = 2$

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(17) If  $Y \propto \sqrt[3]{x}$  and  $Y = \frac{2}{3}$  as  $X = 8$ , Find the value of X at  $Y = 1$

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(18) If Y varies as the multiplicative in verse of the expression  $\frac{1}{x^2}$ , then find the relation between X and Y, if Y = 4 as X = 3, then find the value of Y as X = 9.

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(19) **Complete:**

a) If  $X \propto Y$ , then  $X = \dots\dots\dots$

b) If  $Y \propto X$ , then  $\frac{x_1}{x_2} = \frac{\dots\dots\dots}{\dots\dots\dots}$

c) If  $Y = \frac{3}{5}X$ , then  $Y \propto \dots\dots\dots$

d) If  $X - 2Y = 0$ , then  $X \propto \dots\dots\dots$

e) If  $2XY = 5$  , then  $X \propto$  .....

f) If  $Y \propto X^2$  and  $Y = 5$  as  $X = 1$  , then the constant of variation = .....

# Statistics

## Unit (3)

## **unit (6) Lesson (1)** **Collecting Data**

### **Resources of collecting data:**

#### **1- Primary resources (field resources):**

- \* *get data by interviewing or questionnaires.*
- \* *very accuracy.*
- \* *needs time and efforts.*
- \* *highly expensive.*

#### **2- Secondary resources (historical resources):**

- \* *get data from authorities and agencies.*
- \* *such as central agency for mobilization and statistics.*

\*\*\*\*\*

### **Methods of collecting data:**

#### **1- Method of mass population:**

- \* *collect the data from all the values of the statistical society.*
- \* *ex.: population.*
- \* *It needs long time, great effort and costs much money.*
- \* *It's unbiased and the outcomes are so accurate.*

#### **2- Methods of samples:**

- \* *It depends on selecting a sample from the statistical society.*
- \* *The outcomes we get are generalized on the whole society.*
- \* *It saves time, efforts and money.*
- \* *It's the only way in gigantic societies. (fish)*
- \* *The only way in some limited societies.*
- \* *ex.: check patient blood.*  
*check the validity of lamps.*
- \* *The outcomes are not accurate (biased sample)*

## ***How we select samples and its conditions:***

### **1- The biased selection:** (Not randomly)

- \* We select the sample in a way to satisfy the objectives of the research.
- \* It's called the sample deliberate.
- \* **ex.:**

*If we want to know how the students understood a lesson, we have to choose those who studied this topic of exam.*

### **2- Random selection:** (random sample)

- \* select a sample where the chance of getting any value from the society is equal.
- \* It has two types:

<b><i>(A) Simple random sample:</i></b>	<b><i>(B) Layer random sample:</i></b>
<ul style="list-style-type: none"> <li>* <i>It's the simplest type of samples.</i></li> <li>* <i>It can be get from the homogeneous societies.</i></li> <li>* <i>It selected according the size and no. of units of the society.</i></li> </ul> <p><b><u>(i) The society is small:</u></b></p> <p style="padding-left: 20px;"><i>(Names and no's for all students on identical cards, put in a box and choose randomly).</i></p> <p><b><u>(ii) Use calculator (n° all the students)</u></b></p> <p><i>Start → on → shift → ran # → = → (...)</i></p>	<ul style="list-style-type: none"> <li>* <i>used for heterogeneous society.</i></li> <li>* <i>divide the society into homogenous sets.</i></li> <li>* <i>each set is called a "layer".</i></li> <li>* <i>then select random sample from each layer.</i></li> </ul> <p><b><u>Ex.:</u></b></p> <div style="display: flex; align-items: center; margin: 10px 0;"> <div style="margin-right: 10px;">sample = 50</div> <div style="font-size: 2em;">{</div> <div style="margin-left: 10px;"> <p>30 males</p> <p>20 females</p> </div> </div> <p><i>select with ratio 3 : 2 from each layer.</i></p>

## Algebra 3<sup>rd</sup> Prep.

### Lesson (6-2) "Dispersion"

#### **Central tendency → (mean – median – mode)**

- $mean = \frac{\text{Total of values}}{\text{their number}}, \bar{X} = \frac{\sum x}{n}, \text{mean } \bar{X}$
- $median = \text{middle value after arranging.}$
- $Mode = \text{most repeated number.}$

Ex.: Set A : 5 , 7 , 6 , 2 , 3 , 9 , 3

$$Mean = \frac{5+7+6+2+3+9+3}{7} = 5$$

Set B : 2 , 3 , 3 , 5 , 6 , 7 , 9

Median = 5

Mode = 3

#### Dispersions measurements :

1- Range

2- Standard deviation

#### **1) Range:**

Is the difference between the greatest and the smallest value.

Ex.: 51 , 53 , 55 , 57 , 58 , 60 → set A

$$Range = 60 - 51 = 9$$

Ex.: 42 , 45 , 47 , 49 , 52 , 92 → set B

$$Range = 92 - 42 = 50$$

**Then set (B) is more divergent than set (A).**



**2) standard deviation  $\sigma$  :**

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$n$  : no. of values

$\bar{x}$  : mean

$\sum \dots$  : sum of .....

Ex.: (1)

Find the standard deviation for the values:

15, -12, -9, 27, -6

Answer:      Arrange: -12, -9, -6, 15, 27

$$\bar{x} = \frac{\sum x}{n} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \dots\dots\dots$$

	$X$	$X - \bar{X}$	$(X - \bar{X})^2$
$\Sigma$			

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\quad}{\quad}} = \sqrt{\quad}$$

$\sigma \approx \dots\dots\dots$

**Ex.: (2)**

**Find the standard deviation and the mean for the following table:**

<b><i>Number of defective units</i></b>	<b><i>zero</i></b>	<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>	<b><i>5</i></b>
<b><i>Number of boxes</i></b>	<b><i>3</i></b>	<b><i>16</i></b>	<b><i>17</i></b>	<b><i>25</i></b>	<b><i>20</i></b>	<b><i>19</i></b>

<b><i>Number of defective units</i></b>	<b><i>Number of boxes k</i></b>	<b><i>x × k</i></b>	<b><i>x - <math>\bar{x}</math></i></b>	<b><i>(x - <math>\bar{x}</math>)<sup>2</sup></i></b>	<b><i>(x - <math>\bar{x}</math>)<sup>2</sup> k</i></b>
<b><i>zero</i></b>	<b><i>3</i></b>				
<b><i>1</i></b>	<b><i>16</i></b>				
<b><i>2</i></b>	<b><i>17</i></b>				
<b><i>3</i></b>	<b><i>25</i></b>				
<b><i>4</i></b>	<b><i>20</i></b>				
<b><i>5</i></b>	<b><i>19</i></b>				
<b><i>Σ</i></b>					

***The mean  $\bar{x} = \frac{\sum x \times k}{\sum k} = \text{-----} = \text{.....}$***

***The standard deviation***

***$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2 k}{\sum k}} = \sqrt{\text{-----}} \approx \text{..... units}$***

**Ex.: (3) The following frequency distribution shows the number of children of some families in a new city:**

<b>Number of children</b>	<b>zero</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Number of families</b>	<b>8</b>	<b>16</b>	<b>50</b>	<b>20</b>	<b>6</b>

**Calculate the mean and standard deviation to the number of children.**

<b>Number of children</b>	<b>Number of families <math>k</math></b>	<b><math>x \times k</math></b>	<b><math>x - \bar{x}</math></b>	<b><math>(x - \bar{x})^2</math></b>	<b><math>(x - \bar{x})^2 k</math></b>
<b>zero</b>	<b>8</b>				
<b>1</b>	<b>16</b>				
<b>2</b>	<b>50</b>				
<b>3</b>	<b>20</b>				
<b>4</b>	<b>6</b>				
<b><math>\Sigma</math></b>					

**The mean  $\bar{x} = \frac{\Sigma x \times k}{\Sigma k} = \text{—————} = \text{.....}$**

**The standard deviation :**

$$\sigma = \sqrt{\frac{\Sigma (x - \bar{x})^2 k}{\Sigma k}}$$

$$= \sqrt{\text{—————}} = \text{.....}$$

**Ex. (4) The following frequency distribution shows the weights of 200 students in a school:**

<b>Weight in kg</b>	<b>35-</b>	<b>45-</b>	<b>55-</b>	<b>65-</b>	<b>75-85</b>	<b>Total</b>
<b>Number of students</b>	<b>20</b>	<b>55</b>	<b>80</b>	<b>30</b>	<b>15</b>	<b>200</b>

**Find: The mean and standard deviation of students weights.**

<b>Sets</b>	<b>x</b>	<b>k</b>	<b>x X k</b>	<b>x - <math>\bar{x}</math></b>	<b>(x - <math>\bar{x}</math>)<sup>2</sup></b>	<b>(x - <math>\bar{x}</math>)<sup>2</sup>k</b>
<b>35-</b>		<b>20</b>				
<b>45-</b>		<b>55</b>				
<b>55-</b>		<b>80</b>				
<b>65-</b>		<b>30</b>				
<b>75-</b>		<b>15</b>				
<b><math>\Sigma</math></b>		<b>200</b>				

$$\bar{x} = \frac{\sum x \cdot k}{\sum k} = \frac{\quad}{\quad} = \dots\dots\dots$$

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2 k}{\sum k}} = \dots\dots\dots$$



**(3) The following distribution table shows the amount of gasoline a set of cars consumes:**

<i>No.of km/lit</i>	<i>5-</i>	<i>7-</i>	<i>9-</i>	<i>11-</i>	<i>13-</i>	<i>15-17</i>	<i>Total</i>
<i>No.of cars</i>	<i>3</i>	<i>6</i>	<i>10</i>	<i>12</i>	<i>5</i>	<i>4</i>	<i>40</i>

***Find the standard deviation to the number of kilometers per liter.***

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**(4) Choose the correct answer:**

***1) The difference between the maximum and the minimum value for a set of data represented .....***

***a) The mean***

***b) The standard deviation***

***c) The mode***

***d) The range***

***2) The range for the values 7 , 4 , 9 , 5 and 13 is .....***

***a) 6***

***b) 7***

***c) 9***

***d) 5***

3) *The most repeated value in a set of values represents .....*

*a) mean*

*b) mode*

*c) median*

*d) range*

4) *The range for the set { 51 , 53 , 57 , 52 , 58 , 59 } equals .....*

*a) 8*

*b) 51*

*c) 9*

*d) 59*

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*(5) find the mean and the standard deviation for the values*

*50 , 57 , 100 , 120 and 85 :*

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**(6) Find the mean and the standard deviation for the following data:**

<i>sets</i>	<i>0-</i>	<i>2-</i>	<i>4-</i>	<i>6-</i>	<i>8-</i>
<i>frequency</i>	<i>5</i>	<i>9</i>	<i>15</i>	<i>15</i>	<i>6</i>

[illegible]

(7) A school administrator needs to know the opinions of the students about the students' activities which are offered from the school so he gives each student a number starting from 301 to 700 and selects 5% of them as a random sample from them to ask, uses the calculator to recognize the numbers for the selected students in this sample.

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**(8) If 5 , 6 , 7 , 8 and 9 represent the marks of a pupil in mathematic test in 5 months .find the mean and the standard deviation.**

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**(9) Find the mean and standard deviation of the following distribution:**

<b><i>Number of children</i></b>	<b><i>0</i></b>	<b><i>1</i></b>	<b><i>2</i></b>	<b><i>3</i></b>	<b><i>4</i></b>
<b><i>Number of families</i></b>	<b><i>4</i></b>	<b><i>8</i></b>	<b><i>50</i></b>	<b><i>30</i></b>	<b><i>8</i></b>

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**(10) Find the mean and standard deviation of the following distribution:**

<i>Number of children</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
<i>Number of families</i>	<i>16</i>	<i>50</i>	<i>80</i>	<i>55</i>	<i>20</i>

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**(11) The following distribution for the marks of some students in one of the exams :**

<i>Marks</i>	<i>0-</i>	<i>2-</i>	<i>4-</i>	<i>6-</i>	<i>8-10</i>
<i>Number of students</i>	<i>3</i>	<i>6</i>	<i>10</i>	<i>12</i>	<i>9</i>

**(1) Find the mean**

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**(2) Find the standard deviation for the marks of the students**

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